arXiv:hep-th/0005179v3 24 Oct 2000

INFNCA-TH0009

Two-dimensional black holes as open strings: A new realization of the ADS/CFT correspondence*

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Abstract

We show that weak-coupled two-dimensional dilaton gravity on Anti-de Sitter space can be described by the dynamics of an open string. Neumann and Dirichlet boundary conditions for the string lead to two different realizations of the Anti-de Sitter/Conformal Field Theory correspondence. In particular, in the Dirichlet case the thermodynamical entropy of two-dimensional black holes can be exactly reproduced by counting the string states.

04.70.Dy; 11.25.Pm; 04.50.+h

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*This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement DE-FC02-94ER40818 and by a FCT grant, contract number BPD/20166/99.

The realization of the holographic principle in two spacetime dimensions is a subject that has recently attracted much attention in the literature, where it has been mainly investigated in the context of the Anti-de Sitter/Conformal Field Theory (AdS_d/CFT_{d-1}) correspondence [1]. For $d = 2$ it states that gravity on AdS_2 is dual to a one-dimensional conformal field theory living on the boundary of AdS_2 .

In spite of the efforts that have been devoted to clarify the AdS_2/CFT_1 duality [2–5], the latter remains puzzling and mysterious. Since the conformal symmetry involved in the duality is infinite dimensional, the dynamics is expected to be highly constrained. However, the realization of the symmetry in terms of boundary states is far from trivial [4,5]. This difficulty seems related to the topology of the boundary of AdS_2 , which is one-dimensional and disconnected.

The lack in understanding of the AdS_2/CFT_1 duality has prevented real progress in what is considered its main application: the study of two-dimensional (2D) gravity structures (e.g black holes) [6] using conformal field theory techniques. This application is of fundamental relevance for black holes physics because it can be used to give statistical meaning to the entropy of both 2D black holes and higher dimensional black holes that reduce to 2D models upon compactification. Attempts to calculate the statistical entropy of 2D AdS_2 black holes met only partial success [4,5]. A mismatch of a factor $\sqrt{2}$ between the thermodynamical and statistical entropy was found.

In this letter we clarify the meaning of the AdS/CFT correspondence in two dimensions by showing that it can be realized in two different ways both steming from a more fundamental AdS_2/CFT_2 correspondence. Using the nonlinear sigma model formulation of 2D dilaton gravity [7] we show that weak-coupled dilaton gravity on AdS_2 can be described by the dynamics of an open string. Using Neumann boundary conditions we retrieve the AdS_2/CFT_1 correspondence that has been analyzed in Ref. [4,5]. Dirichlet boundary conditions lead to a new realization of the AdS/CFT correspondence. In this case the properties of 2D black holes have a direct interpretation in terms of string dynamics. In particular, the entropy of the black hole can be exactly computed in terms of the degeneracy of the open string spectrum.

The simplest 2D gravity model admitting AdS_2 as solution is the Jackiw-Teitelboim (JT) model [8]

$$A = \frac{1}{2}\int \sqrt{-g}\, d^2x\, \phi\left(R + 2\lambda^2\right) . \tag{1}$$

The classical solutions of the model,

$$ds^2 = -\left(\lambda^2 r^2 - \frac{2m}{\lambda\phi_0}\right)dt^2 + \left(\lambda^2 r^2 - \frac{2m}{\lambda\phi_0}\right)^{-1} dr^2\,, \quad \phi = \phi_0\lambda r\,, \quad m \geq 0\,, \tag{2}$$

can be interpreted as AdS_2 -black holes [9]. The black hole mass m appearing in Eq. (2) is defined by the mass functional

$$M = \lambda^2\phi^2 - \partial_\rho\phi\partial^\rho\phi. \tag{3}$$

On-shell M is constant [10] and equal to $2\phi_0\lambda m$.

The 2D gravity model (1) is pure gauge, i.e., it has no physical local degrees of freedom. Moreover, solutions (2) with different values of m represent different, locally equivalent,

parametrization of AdS_2 . However, the presence of the scalar ϕ makes them globally nonequivalent [9]. Following the notation of Ref. [9] we will denote with AdS_2^+ and AdS_2^0 the black hole solutions ($m > 0$) and the ground state ($m = 0$), respectively.

The link between 2D AdS-gravity and CFT can be established using the asymptotic symmetries of AdS_2 . It has been shown in Refs. [4,5] that the asymptotic symmetries of AdS_2 are generated by a Virasoro algebra and that the deformations of the timelike boundary of AdS_2 give a realization of the conformal symmetry. In Refs. [4,5] the (r, t) coordinates of Eq. (2) have been used to discuss the asymptotic symmetries of AdS_2 , yet for our purposes it is convenient to use light-cone coordinates (u, v). In the (u, v)-frame the AdS_2^0 solution is $g_{uv} = 2/\lambda^2(u+v)^2$, $g_{uu} = g_{vv} = 0$, $\phi = -\phi_0/\lambda(u+v)$, so the boundary conditions [4] to be imposed on the metric and on the dilaton are $g_{uv} = 2/\lambda^2(u+v)^2 + O(1)$, $g_{uu} = O(1)$, $g_{vv} = O(1)$, and $\phi = O((u+v)^{-1})$, respectively. The metric and the dilaton have the asymptotic, $u \to -v$, form

$$
\begin{aligned}
g_{uu} &= U_0 + \ldots + U_n(u+v)^n + \ldots\,, \\
g_{uv} &= \frac{2}{\lambda^2(u+v)^2} + Y_0 + \ldots + Y_n(u+v)^n + \ldots\,, \\
g_{vv} &= V_0 + \ldots + V_n(u+v)^n + \ldots\,, \\
\phi &= -\phi_0\left[\frac{\omega_{-1}}{\lambda(u+v)} + \omega_1\lambda(u+v) + \ldots + \omega_n\lambda^n(u+v)^n + \ldots\right],
\end{aligned}
\tag{4}
$$

where the coefficients U_k, U_k, ω_k are functions of $u - v$ only. The transformations generated by the asymptotic symmetry group leave unchanged the leading terms in Eq. (4) and act on the remaining functions U_k, V_k, Y_k and ω_k. These can be thought as characterizing the deformations of the $u = -v$ boundary of AdS_2 .

The asymptotic symmetry group is generated by the Killing vectors

$$
\begin{aligned}
\chi^u &= \frac{1}{2}\left[\epsilon + \epsilon'(u+v) + \frac{1}{2}\epsilon''(u+v)^2\right] + \alpha^u\,, \\
\chi^v &= \frac{1}{2}\left[-\epsilon + \epsilon'(u+v) - \frac{1}{2}\epsilon''(u+v)^2\right] + \alpha^v\,,
\end{aligned}
\tag{5}
$$

where $\epsilon \equiv \epsilon(u - v)$ is an arbitrary function, $\alpha^{u,v} = \sum_{k=3}^{\infty} \alpha_k^{u,v}(u-v)(u+v)^k$, and $' = d/d(u-v)$. $\alpha^{u,v}$ represent "pure gauge" diffeomorphisms of the 2D gravity theory that fall off rapidly as $u \to -v$. The Killing vectors (5) define a conformal group generated by a Virasoro algebra and the boundary fields $\Theta_k = (U_k, V_k, Y_k, \omega_k)$ span a representation of this symmetry. Their transformation law has the form

$$
\delta_\epsilon \Theta_k = \epsilon\Theta_k' + (h+k)\epsilon'\Theta_k + \ldots\,,
\tag{6}
$$

where $h = 2$ for the fields U, V, Y and $h = 0$ for the fields ω, and dots denote terms that depend on higher derivatives of ϵ and on the "pure gauge" diffeomorphisms. It is important to notice that although "pure gauge" transformations affect the boundary fields, the charges that are associated with the asymptotic symmetry are invariant [5] under pure gauge transformations. The mass functional M can be likewise expanded near the $u = -v$ boundary

$$M = \sum_{k=0}^{\infty} M_k (u - v)(u + v)^k \,. \tag{7}$$

Using Eqs. (3) and (4) the M_k can be expressed in terms of the boundary fields. They follow the transformation law (6) with $h = 0$.

The action (1) can be cast in the form of a 2D conformal nonlinear sigma model [7] with Lagrangian

$$\mathcal{L} = \sqrt{-g}\partial_\mu M \partial^\mu \psi \cdot \frac{1}{1 - 4\lambda^2\psi^2 M} \,, \tag{8}$$

where $\phi = (-2\lambda^2\psi)^{-1}$ and we have neglected irrelevant surface terms. The Lagrangian (8) can be expanded around $\psi = 0$

$$\mathcal{L} = \sqrt{-g} \sum_{k=0}^{\infty} \partial_\mu M_k \partial^\mu \psi_k \,, \tag{9}$$

where $M_k = M^{k+1}/(k+1)$, $\psi_k = (2\lambda\psi)^{2k+1}/2\lambda(2k+1)$. Equation (9) is both a perturbative expansion in terms of the (coordinate-dependent) gravitational coupling of the model (1) (ϕ^{-1}) and an expansion near the boundary of AdS_2 . Each term in Eq. (9) has the form of a free-field conformal theory and transforms according to Eq. (6) with $h = 2$, although the sum (full theory) does not. In the weak-coupling regime, $\psi << 1$, the theory can be treated perturbatively. In particular, if we restrict ourselves to the first order in the perturbative expansion the theory reduces to a free CFT and the usual machinery of CFTs can be applied. In this sense we speak of "duality" between (weak-coupled) AdS_2 gravity and CFT. Let us stress that the identification of a weak-coupling regime, where a consistent perturbative expansion can be constructed, is a fundamental feature of the sigma model representation that does not have counterpart in the original formulation based on Eq. (1) and is essential for the identification of the fundamental microscopic degrees of freedom of the theory. In this paper we shall focus attention on the first term in the perturbative expansion (9) leaving the discussion of higher terms to further investigations. This amounts to neglecting, in first approximation, higher order perturbative corrections generated by the running gravitational coupling. Note that the latter becomes strong near $r = 0$, i.e., precisely in the "opposite" region of AdS_2 around which we are expanding. In this approach higher order corrections are described by interacting terms for the free CFT.

It is convenient to define the new fields

$$\begin{aligned} \sqrt{\pi\alpha'} M_0 &= \frac{1}{2}(X^1 + iX^2) = \frac{1}{2}(X^0 + X^1) \,, \\ \sqrt{\pi\alpha'} \psi_0 &= \frac{1}{2}(X^1 - iX^2) = \frac{1}{2}(X^1 - X^0) \,, \end{aligned} \tag{10}$$

where α' is a constant with dimension of $lenght^2$. Using complex coordinates $z \equiv u = (t + x)/2 = (\sigma^1 + i\sigma^2)/2$ and $\bar{z} \equiv v = (x - t)/2 = (\sigma^1 - i\sigma^2)/2$, the leading term in the expansion (9) can be cast in the usual bosonic string form. [See Ref. [11] for notations]. Since AdS_2 has a timelike boundary at $x = 0$, we are dealing with open strings and the expansion (9) defines a $\mathrm{AdS}_2/\mathrm{CFT}_2$ correspondence between open string theory and dilaton gravity on AdS_2. Boundary conditions are restricted to Dirichlet ($X^\mu(x = 0) = const$) or Neumann

($n^a \partial_a X^\mu(x=0) = 0$) type, respectively. Mixed boundary conditions are not allowed because from the boundary expansion (4) for the field ϕ it follows $\partial_t X^0(x=0) = \partial_t X^1(x=0)$. The choice of boundary conditions determines the realization of the AdS/CFT correspondence. The $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence that has been proposed in Ref. [4] is obtained by imposing Neumann boundary conditions, which allow for excitations on the boundary. In this case we have $X^\mu(x=0) = F(t) = M_0$ and the conformal symmetry can be realized on the boundary by the charges that are associated with the asymptotic symmetries of AdS_2 . Dirichlet boundary conditions break translation invariance in the x direction and no dynamical degrees of freedom are allowed on the boundary, the string endpoint being fixed. In this case we are naturally lead to a new realization of the AdS/CFT correspondence. It is shown below that the correspondence is realized in terms of pure deformations of the boundary of AdS_2.

In addition to the timelike boundary at $x=0$, AdS_2^0 has an inner null boundary. However, the presence of the latter does not influence the dynamics of the open string. Writing the solution, Eq. (2), in the conformal coordinate frame (t,x), one finds that AdS_2^0 is conformal to Minkowski space and that the presence of the dilaton requires

$$-\infty < t < \infty\,, \qquad 0 \le x < \infty\,. \tag{11}$$

In this coordinate frame the inner null boundary is located at $x=\infty$. Hence, because of conformal invariance, open strings on AdS_2^0 are equivalent to open strings on the region of the Minkowski spacetime defined by Eq. (11).

The $\mathrm{AdS}_2/\mathrm{CFT}_2$ correspondence is expressed in a exact form by putting in a one-to-one correspondence the symmetries and local degrees of freedom of the open string and the asymptotic symmetries and excitations of AdS_2 . The conformal symmetry in two spacetime dimensions is generated by the Killing vectors, $\chi = \chi(z)\partial + \bar\chi(\bar z)\bar\partial$. A generic CFT_2 field $\psi(z,\bar z)$ of weights $(h,\bar h)$ transforms as

$$\delta_{\chi,\bar\chi}\psi = (\chi\partial + h\partial\chi)\psi + (\bar\chi\bar\partial + \bar h\bar\partial\bar\chi)\psi\,. \tag{12}$$

Dirichlet boundary conditions require that χ and $\bar\chi$ are related by the condition

$$\chi(z) + \bar\chi(\bar z) = 0\,. \tag{13}$$

This equation implies that the conformal symmetry is generated by a single copy of the Virasoro algebra. By expanding the Killing vectors on the boundary we obtain Eq. (5) with $\chi((z-\bar z)/2) = -\bar\chi(-(z-\bar z)/2) = \epsilon(u-v)/2$, where the pure gauge diffeomorphisms have been fixed as $\alpha_k^u = (-1)^{k+1}\alpha_k^v = (1/2k!)d^k\epsilon/d(u-v)^k$. Thus, by fixing the pure gauge diffeomorphisms appropriately, the symmetry group of the Dirichlet open string and the asymptotic symmetry group of AdS_2 coincide. Each AdS_2 field living near $x=0$ can be interpreted as the coefficient of the expansion of the CFT_2 field around the boundary with given weight $h+\bar h$ and pole of order p. Moreover, the above correspondence allows to determine from CFT_2 the Virasoro generators of the asymptotic symmetry group of AdS_2. Using Eq. (13) and expressing the CFT_2 Virasoro generators $L_m^{CFT} = z^{-m+1}\partial$ and $\tilde L_m^{CFT} = \bar z^{-m+1}\bar\partial$ as functions of the x,t coordinates we find

$$L_m^{AdS} = 2^{m-1}\left\{\left[(t+x)^{-m+1} + (t-x)^{-m+1}\right]\partial_t + \left[(t+x)^{-m+1} - (t-x)^{-m+1}\right]\partial_x\right\}\,. \tag{14}$$

The AdS Virasoro generators (14) are valid both on the boundary and outside the boundary, where they generate the full symmetry group of the open string with Dirichlet boundary conditions. Equation (14) leads to the asymptotic AdS Killing vectors (5) with fixed gauge diffeomorphisms. By fixing the pure gauge diffeomorphisms of the AdS asymptotic symmetries we can reconstruct the full symmetry group of the Dirichlet open string. According to this picture the Virasoro generators L_m^{AdS} cannot be interpreted as generating the symmetries of a 1D conformal field theory living on the boundary of AdS_2 , the latter being frozen by the Dirichlet boundary conditions.

The $\mathrm{AdS}_2/\mathrm{CFT}_2$ correspondence can also be realized using local oscillator degrees of freedom. Let us expand the string field in normal modes

$$X^\mu = x^\mu - ip^\mu \log|z|^2 + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{\infty} \frac{1}{m}\left(\alpha_m^\mu z^{-m} + \tilde{\alpha}_m^\mu \bar{z}^{-m}\right) . \tag{15}$$

Comparing Eq. (15) to the asymptotic expansions of M_0 and ψ_0

$$M_0 = \sum_{k=0}^{\infty}\sum_{m=-\infty}^{\infty} M_{km} x^k t^m , \qquad \psi_0 = \sum_{k=1}^{\infty}\sum_{m=-\infty}^{\infty} \Psi_{km} x^k t^m , \tag{16}$$

we find [we assume $t > 0$ for simplicity]

$$\alpha_m^\mu = (-1)^{m+1}\tilde{\alpha}_m^\mu = i\sqrt{\pi}2^{-1/2-m}\left[M_{1,-1-m} \mp \Psi_{1,-1-m}\right] , \tag{17}$$

where we have imposed the Dirichlet boundary conditions that imply $p^\mu = 0$, $M_{00} = const$, and $M_{0m} = 0$ for $m \neq 0$. The asymptotic excitations of the gravity theory are in a one-to-one correspondence with the open string modes. Moreover, the lower terms in the asymptotic expansion are sufficient to determine the whole CFT_2 theory. The fields M_1, Ψ_1 and M_0 are invariant under pure gauge bulk transformations and transform conformally with weight $h = 1$ (M_1 and Ψ_1) and $h = 0$ (M_0). Therefore, Eq. (17) provides a realization of the $\mathrm{AdS}_2/\mathrm{CFT}_2$ correspondence: Asymptotic 2D gravity modes around the boundary that describe boundary deformations determine completely CFT_2 (the open string theory) and vice versa. Finally, using Eq. (17) the CFT_2 Virasoro generators can be expressed in terms of the asymptotic modes

$$L_m^{CFT} = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{m-n}^\mu \alpha_{\mu n} = -\pi 2^{-m} \sum_{n=-\infty}^{\infty} M_{1,-1-n}\Psi_{1,-1-m+n} . \tag{18}$$

By imposing Neumann boundary conditions on the open string, the string modes are determined by the gravitational modes $M_{0,m}$. In this case the Virasoro generators (18) and the CFT_2 action are zero at any order in the expansion and the conformal symmetry cannot be realized in terms of local string oscillators. Rather, we are dealing with a topological theory which has no physical local degrees of freedom and the conformal symmetry is realized by the charges associated with the asymptotic symmetries [4].

The $\mathrm{AdS}_2/\mathrm{CFT}_2$ correspondence leads to a natural interpretation of the Hawking evaporation of the AdS_2 black hole [9]. From Eq. (14) it follows that the invariant $SL(2,R)$ algebra is generated by

$$L_0^{AdS} = t\partial_t + x\partial_x , \qquad L_1^{AdS} = 2\partial_t , \qquad L_{-1}^{AdS} = \frac{1}{2}(t^2 + x^2)\partial_t + xt\partial_x . \tag{19}$$

In the representation above L_0^{AdS} does not generate translations in t but dilatations. L_0^{AdS} generates time translations in the T, X coordinates

$$\lambda t = e^{\lambda T} \cosh(\lambda X) \,, \qquad \lambda x = e^{\lambda T} \sinh(\lambda X) \,. \tag{20}$$

Using Eq. (20) the metric of the AdS_2^0 ground state is

$$ds^2 = \frac{1}{\sinh^2(\lambda X)} (-dT^2 + dX^2) \,. \tag{21}$$

Equation (21) describes a 2D black hole [9]. Hence, the Hawking evaporation process can be explained in the CFT_2 context using the same arguments of Ref. [9]. Positive frequency modes of a quantum field with respect to Killing vector ∂_t are not positive frequency modes with respect to Killing vector ∂_T, i.e., the vacuum state for an observer in the (X, T) reference frame appears filled with thermal radiation to an observer in the (x, t) frame. The value of the Hawking flux has been calculated in Ref. [9].

The correspondence between the open string with Dirichlet boundary conditions and the 2D AdS_2 black hole can be used to calculate the statistical entropy of the latter. Since local oscillators of the Dirichlet string are in one-to-one correspondence with excitations of AdS_2^0 we can count black hole states by counting states of CFT_2. To this purpose, we calculate the central charge c associated with the central extension of the Virasoro algebra generated by L_m^{CFT}. Keeping in mind the interpretation of c as a Casimir energy (see for instance Ref. [11]), the transformation law of the stress-energy tensor under the change of coordinates (20) ($w = T + X$) is

$$T^{(2)}_{ww} = (\partial_w z)^2 T^{(2)}_{zz} - \frac{c}{12} \{w, z\} (\partial_w z)^2 \,. \tag{22}$$

The vacuum energy is shifted by $l_0^{(2)} \to l_0^{(2)} - \frac{c}{24}$, where $l_0^{(2)}$ is the eigenvalue of L_0^{CFT} which is associated to the vacuum. This shift corresponds to a Casimir energy $E = -\frac{c}{24}\lambda$.

The coordinate transformation (20) maps the AdS_2^0 ground state solution of the 2D dilaton gravity theory into the AdS_2^+ black hole solution (21) with mass $m = \frac{\phi_0}{2}\lambda$ (see Ref. [9]). Because of the correspondence between the gravitational theory and the Dirichlet string we can interpret the previous map as the gravity theory counterpart of the shift of L_0^{CFT} in CFT_2 and equate the Casimir energy E with m. There is a subtlety concerning the sign to be used in the equation. The coordinate transformation (20) is analogous to the coordinate transformation that maps the Rindler spacetime into the Minkowski spacetime, i.e., it maps *observers*. So an observer in the AdS_2^+ vacuum sees the AdS_2^0 vacuum as filled with thermal radiation with *negative* flux [9]. Since the Casimir energy E is the energy of the z-vacuum as seen in the w-frame, we must use the equation $E = -m$ which leads to $c = 12\phi_0$. Finally, the eigenvalue of L_0^{CFT} can be expressed in terms of the black hole mass. Using the Cardy formula [12] the statistical black hole entropy is

$$S = 2\pi \sqrt{\frac{c L_0^{CFT}}{6}} = 4\pi \sqrt{\frac{\phi_0 m}{2\lambda}} \,, \tag{23}$$

in complete agreement with the thermodynamical result.

In this letter we have proved that the correspondence between 2D gravity and open strings allows for two distinct realization of the AdS/CFT correspondence. The first realization, which is obtained by imposing Neumann boundary conditions to the open string, implies the existence of a genuine one-dimensional CFT living on the boundary of AdS_2 . This realization is, however, problematic from different points of view [4,5]. The realization which is obtained by imposing Dirichlet boundary conditions supports the viewpoint of Ref. [13], where, by quite a different argument, the authors conclude that the correspondence should be realized as AdS_2 /CFT_2, rather than AdS_2 /CFT_1.

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